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HOT-WIRE HEAT LOSS AND FLUCTUATION SENSITIVITY FOR INCOMPRESSIBLE FLOW

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ABSTRACT

An experimental study of the forced convection heat loss from long thin cylinders placed normal to an incompressible flow has been carried out. Tests were made in three different gases (air, argon, helium) at atmospheric pressure and in air over a pressure range 0.2 to 8 atm. Particular attention was given to determining the effect of cylinder temperature on the heat loss. It was found that the "constants" employed in the usual empirical representation of this heat loss vary with cylinder temperature in a way dependent on Knudsen number, results in helium giving a marked indication of this. A velocity fluctuation sensitivity equation was derived which includes the effects of this temperature variation.

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I. INTRODUCTION

Although the hot-wire anemometer has been employed both as an AC and a DC instrument for more than thirty years, the general relation between the heat loss from the wire and the properties of the stream passing over it remains uncertain. Neither precise theoretical formulae nor general experimental data for the total heat loss from long cylinders placed normal to a stream at low but finite Reynolds numbers are available. The few theoretical treatments of the problem suffer from the imposition of unrealistic simplifications; the experimental studies from either lack of precision or insufficient generality. Only one valid limiting case has been solved theoretically. It was given in simplest form comparatively recently by Cole and Roshko (1954). In this solution, Oseen flow was assumed; the momentum and energy equations were thereby rendered independent and the latter was solved for the total heat loss from an infinite cylinder.* The theory is valid only for the limiting case $R \to 0$; in addition, since a constant thermal conductivity was assumed, it applies only for small temperature differences between the cylinder and the stream. Experiment (Collis and Williams, 1959) indicates an approach to the theoretical result for very small Reynolds numbers. An extension of this theory to include temperature-jump effects for near-continuum flow about the cylinder has been given (Levey, 1959) but its verification requires a knowledge

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* For reference to other Oseen-type solutions, see Corrsin (1961).

of the thermal accomodation coefficient which is generally difficult to obtain.

For Reynolds numbers of the order of unity or greater, the form of the heat loss law has conventionally been assumed to be that of the empirical King's law (King, 1914), which, expressed in non-dimensional form, is

$$Nu = A + B\sqrt{R}$$

where A and B are considered as constants, Nu is the wire Nusselt number, R the Reynolds number. A variety of reference temperatures upon which to base the conductivity, density, and viscosity have been employed in this law. While the values of the quantities A and B reported by various investigators are diverse, usage of the square root of the Reynolds number is fairly uniform (Morkovin, 1956, p. 22; Kovasznay, 1954, p. 241)*. A notable exception is the careful series of tests reported recently by Collis and Williams (1959). There, a Reynolds number exponent of 0.45 was found to fit the data best in the range $0.02 < R < 44$.

Although there has been considerable experimental emphasis recently on hot-wire heat loss in compressible flow, the many details of the incompressible case which have not yet been adequately explored leave ample room for study. The effect of wire temperature on the heat loss characteristics is uncertain, heat loss measurements in gases other than air are scarce, and the effect of Knudsen number on incompressible heat loss is virtually unexplored. A knowledge of

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* The explanation often given for this usage is the square-root dependence with Reynolds number of the heat loss in boundary layer problems; such a rationalization is, however, difficult to justify for the low Reynolds numbers commonly employed in hot-wire anemometry.

these effects is not only of interest in itself but is crucial for precise operation of the hot-wire as a fluctuation-sensing device. In this paper, a series of heat loss measurements conducted in three gases (air, argon, helium) and at several ambient density levels, with special attention to the effect of wire temperature, is first presented. An analysis of the fluctuation sensitivity of the hot-wire based on these heat loss results follows.

In view of the divergence in opinion over the proper Reynolds number exponent to use, the more general heat loss law,

$$Nu = A + BR^n \tag{1}$$

has been assumed for fitting the data obtained. The wires were operated at a series of wire temperatures and the possible temperature dependence of n as well as A and B was explored by least-squares fitting to the general form (1) of the set of Nu vs. R data obtained at each wire temperature. The reference temperature upon which to base the fluid properties was arbitrarily taken to be the usual film temperature (mean of wire and free stream temperatures) since there is some evidence that the temperature effect is reduced by using this base. Conversion to other reference temperatures is, however, a simple matter.

II. APPARATUS

A. Tunnel

For all the measurements reported herein, the variable density low speed blow down tunnel of the Gas Dynamics Laboratory was employed. This tunnel operates over a range of 0.1 to 10 atmospheres ambient pressure level. The piping arrangement for the supply of air to the tunnel allows use of either banks of 3,000 psi air stored external to the laboratory or of an auxiliary regulated air supply at a considerably lower pressure. For operation with gases other than air, a simple T-connection permits the air supply to be shut off and other gases to be introduced. These gases were stored in external pressure regulated trallers; they had a specified purity of 99.995% and a moisture content of less than 0.02 mg/liter. No information is available on the properties of the air used; however, some drying of the air is accomplished prior to its storage in the external tanks. In the case of the low density runs, the tunnel exit was connected to the air ejector system of the laboratory.

The free stream turbulence level of the tunnel was sufficiently low so as not to influence any of the heat loss results. A porous steel filter of 35 micron pore size was installed at the tunnel inlet; it virtually eliminated the problems of hot wire breakage and dirt accumulation. To measure high static pressure, a Bourdon-type absolute gage with range 0-200 inches mercury and scale divisions of 0.1 inches mercury was used. This gage had been previously calibrated and the appropriate corrections were applied in each case to the indicated pressures. For still higher pressures, a gage with range

0-200 psi and scale divisions of 0.5 psi was employed. In the low pressure tests, a bank of U-tube mercury manometers was used. One end of the manometer tubes was left open to the atmosphere, and the atmospheric pressure recorded from a wall-mounted mercury barometer. Since the running time was limited in these tests due to the high mass flow rate of the ejector, all the pressure data were taken on film by means of a recording camera. A total head pitot-probe in conjunction with an inclined-tube manometer was employed to measure the mean flow velocity in the tunnel. The manometer had a range of 0-2 inches of manometer fluid with 0.01-inch scale divisions.

B. Hot-Wires

The physical and geometric properties of the wires employed are summarized in the following table:

Table I: Wire Properties

Wire Material	Diameter, d (in.$\times 10^4$)	First coeff. of Resistance α_o ($^oC^{-1}\times 10^3$)	Second coeff. of Resistance β_o ($^oC^{-2}\times 10^7$)	Conductivity k_w (watts/cm^{o}C)	Relative Density S_w	Heat Capacity c_w (cal/gm)	Length ℓ (in.)
90Pt-10Rh	1.0 2.5	1.7	-1.0(?)	0.30	21.4	0.035	0.08
Pt	1.0	3.6	-7.0(?)	0.75	21.4	0.035	0.08

The data in this table are a combination of nominal values given by the manufacturer (composition, diameter), reference values taken from either handbooks or other sources (β_o, k_w, S_w, c_w), and measured values averaged over all the wires used (ℓ, α_o).

The manufacturer of all the wires was the Sigmund Cohn Company, Mount Vernon, New York. The nominal value of diameter given by the supplier has been found by other workers to be fairly accurate.

Lawrence and Landes (1952) report a series of electron-microphotographs taken of various samples supplied by the same manufacturer. The agreement in diameter is generally about 5% with some variation from sample to sample.

The reference quantities given in the table above have been selected by averaging these properties over the slightly conflicting values which are reported from various sources. In general, the agreement between these sources is of the order of 5%. The exception is the value of the "second temperature coefficient of resistance", β_o, which occurs as the quadratic coefficient in the empirical fitting of wire resistance vs. temperature given in (24). Since for moderate temperatures and pure substances, the resistance is closely linear with temperature, the value of β_o is usually small and is therefore difficult to measure precisely. In addition, β_o seems to vary somewhat with the temperature-history of the wire and with the actual size of the wire.

The values given as measured in the table are average values for the wires for which such measurements were taken. The measurement of α_o was accomplished by inserting the wire in an insulated container and recording the resistance as a function of ambient temperature. The values of ℓ were measured by a travelling microscope.

The wire supports were common steel sewing needles. A high resistance sealer, de Khotinsky cement, was used to fix the probe leads and needles to a ceramic cylinder. A typical probe head is shown in Figure 1. Two of the actual wires employed are shown in Figure 2. The wires were etched from larger Wollaston wires which had been soldered to the needle tips.

III. EXPERIMENTAL PROCEDURE

A. Cold Resistance

As is customary, the wire comprised one arm of a Wheatstone bridge. The heated wire resistance was set to a predetermined value by adjusting the bridge current and the current was measured by applying a potentiometer across a precision resistor in series with the wire.

In order to set the wire temperature at a fixed value, it is first necessary to know the resistance of the unheated wire at the ambient temperature of the tunnel. Since the resistance is measured by a bridge arrangement, some current must be passed through the wire. This current will, of course, induce some amount of heating of the wire and consequently a method must be devised to extrapolate the resistance measured back to a condition of zero heating. The conventional method used is as follows:

1) With a flow of fixed velocity over the wire, a small known current, i_1 (say 2 ma), is passed through the wire, and the wire resistance, R_1, is measured.

2) A current $i_2 = \sqrt{2}\ i_1$ is then passed through the wire and the wire resistance R_2 measured.

3) The unheated wire resistance may then be computed as

$$R_o = 2\ R_1 - R_2 \quad . \tag{2}$$

The relation (2) may be easily shown. The heat loss law (1) for wire temperatures close to ambient is, in dimensional form,

$$i^2 \sim (R-R_o)\ (1+cU^n)$$

where c is a constant and U is the stream velocity. Thus for two different currents at fixed velocity,

$$\frac{I^2_1}{I^2_2} \cong \frac{R_1-R_o}{R_2-R_o}$$

If $I_2 = \sqrt{2}\ I_1$, then (2) is immediately obtained.

It is, of course, necessary to subtract the lead resistance from the measured values. The lead resistance was obtained by inserting the unetched hot-wire probe into the circuit and taking the resistance measured to be the lead resistance. This procedure was accurate since the contribution of the resistance of the unetched portion of the wire to the total lead resistance was negligible.

In many cases, the hot-wire filaments underwent considerable cold resistance changes during initial testing. This phenomenon has been noted by many investigators. The initial rapid decrement observed in the resistance is probably due to the annealing effect of high wire temperatures which relieves the cold-working imposed by the drawing process used in manufacture. Subsequently, however, the wire resistance usually slowly increased. A satisfactory explanation of this increase has not been given. It may be a result of the purging of adsorbed gas or other impurities in the wire. To stabilize the cold resistance, the wire was operated at the maximum temperature to be used in the measurements and the heated resistance monitored. After some length of time, which varied from minutes to hours among wire samples, the resistance usually stabilized and in subsequent tests remained nearly constant.

B. Heat Loss

To measure the flow velocity, a pitot tube was stationed close to the wire. At a constant wire temperature, the mass flow over the wire was varied and the wire current and the dynamic pressure measured. Another static tap connected to an appropriate manometer yielded the absolute pressure level. In the low density tests, the pressure level of the tunnel was held fixed and the velocity alone varied. This was made possible by means of a variable-area choking nozzle at the tunnel exit. These tests were therefore performed at constant wire Knudsen number.

All of the tests were conducted over a range of wire Reynolds numbers (where the viscosity μ , and the density ρ, are based on film temperature) of about

$$0.4 < R < 4$$

This restricted range was used to avoid possible changes in the heat loss characteristics due to the appearance of discrete vortices on the leeward side of the wire. The Reynolds number at which these form is generally believed to be about 4.

Considerable care was exercised to insure that the ambient stream temperature remained constant during the heat loss tests. Variations of as much as $1^{\circ}F$. in this temperature during a run were uncommon. All of the electrical and aerodynamic quantities measured were checked from time to time by independent measurement using alternate equipment; excellent agreement was always found.

IV. DATA REDUCTION

A. General Remarks

Preliminary evaluation of the data indicated that at least 8 evenly distributed data points for each test were required, and preferably 10 or 12, for consistency. Consequently, all tests having fewer points than 8 were not considered in the evaluation of the heat loss data and if any discrepancies occurred among tests under the same operating conditions, preference was given to tests containing the most number of points. Discrepancies among tests with 10 or more points were rare and usually traceable to some gross procedural error.

The reduction of all data was carried out on an IBM 650 electronic digital computer. The use of such a computer is virtually mandatory for least-squares data fitting of the numerous heat loss tests carried out.

For the density, the perfect gas equation of state was assumed. The mean free path was computed from (Kennard, 1938)

$$4) \qquad \lambda = \nu \sqrt{\pi/2R_gT} \quad ,$$

(where $\nu = \mu/\rho$ and R_g is the gas constant), and was evaluated at the wire temperature and in some cases at the reference temperature.

The physical properties of the wires used are given in Table I. Since the principal emphasis in this work is on determining the temperature dependence of A, B, n as well as on testing in several gases at various Knudsen numbers, no special effort was made toward fixing precisely the absolute values of A, B. Accuracy in these quantities depends principally upon accurate (and frequent) determination of $\alpha_0 R_0/\ell$ and wire diameter. The consistency of the results using

averaged values for these quantities in the data reduction is the only available method of estimating errors due to their variation. Sample measurements of α_o agree well with values reported elsewhere, however, the range of temperatures employed was insufficient to determine precisely the value of the second coefficient, β_o. Since β_o is at any rate expected to be small for Pt-Rh wires, it was taken to be zero in the computations. A discussion of the possible error introduced by this assumption is given later.

B. Computation Formulae

Nusselt number: The general definition of Nusselt number is

$$5) \qquad Nu = \frac{hd}{k} \quad ,$$

where the film coefficient, h, is

$$h = \frac{q}{A_w(T_w - T_o)}$$

and q is the total heat transfer, A_w the wire surface area. Thus for an ohmically heated infinite cylinder,

$$6) \qquad Nu = \frac{I^2 R_w}{\pi \ell k (T_w - T_o)} \quad .$$

If the conductivity is evaluated at the film temperature

$$T_f = \frac{T_w + T_o}{2}$$

then assuming a linear resistance-temperature relation,

7) $$Nu = \frac{I^2}{a_w} \frac{1+a_w}{1+ha_w} \frac{\alpha_o R_o/\ell}{\pi k_o} \quad ;$$

where

$$a_w = \frac{R_w - R_o}{R_o}$$

is the so-called "overheating ratio" and

$$h = \frac{(dk/dT)_m}{2k_o\alpha_o}$$

For purposes of data reduction, it is more convenient to use a_w as a measure of wire temperature rather than the temperature loading quantity

$$\tau = \frac{T_w - T_o}{T_o} \quad .$$

Corrections: To account for loss of heat to the wire supports by conduction the approximate relation for an "infinite wire" Nusselt number,

8) $$N_u^+ = \frac{Nu}{1+1.1S/\sqrt{1+a_w}} \quad ,$$

given by Kovasznay (1954) was used, where

9) $$S = \frac{d}{\ell}\sqrt{\frac{k_w}{Nuk}} \quad .$$

Betchov (1952) has studied the end loss correction when the quantity $A = A(\tau)$ and it may be shown from his results that, to order S, the effect of end loss and variable A are independent. The correction to

order S is given by (8) except that the factor 1.1 is replaced by unity. This factor was used by Kovasznay to give a slightly better approximation to the end loss correction while retaining only the first order term in S.

A method of computing the effect of "temperature-jump" in near continuum flow, suggested by Collis and Williams (1959) and found theoretically by Levey (1959), simply involves correcting the temperature of the gas at the wire surface by the amount predicted for the temperature jump from elementary kinetic theory. It yields for the continuum Nusselt number

$$N_u^{\times} = \frac{N_u}{1-CK_nN_u} \tag{10}$$

where

$$C = \frac{2(2-f)}{f} \; \frac{\gamma}{\gamma+1} \; \frac{1}{Pr} \tag{11}$$

and f, γ, Pr are the thermal accomodation coefficient, specific heat ratio, and Prandtl number, respectively. The applicability of this simple correction outside the range $R \to 0$ remains to be demonstrated; however, for want of a more precise method, it was used in the data reduction here. Even assuming its validity, there is considerable question about what value to use for the accomodation coefficient, f. As pointed out by Collis & Williams, substitution of a typical value of 0.9 reported in the literature for air on platinum gives a value of C for air very close to 2 which was consequently employed in their data reduction. The same value was used here for both air and argon on both Pt and 90Pt-10Rh wires. The case of helium is discussed later.

Combining the two corrections above gives

$$\text{(12)} \qquad Nu^* = \frac{N_u}{(1 - CK_n N_u)\ (1 + 1.1S/\sqrt{1 + a_w}}$$

as the continuum infinite-wire Nusselt number.

Reynolds number: The wire Reynolds number is

$$R = \frac{\rho U d}{\mu} \quad .$$

Based on film temperature, this becomes

$$\text{(13)} \qquad R = \frac{\rho_o U d}{\mu_o} (1 + h_1 a_w)\ (1 + h_2 a_w),$$

where

$$h_1 = \frac{(d\mu/dT)_m}{2\mu_o \alpha_o} \quad , \quad h_2 = \frac{1}{2\alpha_o T_o} \quad .$$

Knudsen number: The wire Knudsen number is

$$K_n = \frac{\lambda}{d}$$

and using equation (4),

$$K_n = \frac{\nu}{d} \sqrt{\pi/2R_g T} \quad .$$

Since non-continuum effects probably depend on the properties at the wire rather than in the external field, these properties were usually evaluated at the wire temperature, giving

$$\text{(14)} \qquad K_n = \frac{\nu_o}{d} \sqrt{\pi/2R_g T_o} \quad (1 + 2h_1 a_w)\ (1 + 2h_2 a_w)^{\frac{1}{2}}$$

Estimates of radiation and free convection effects indicated that these were entirely negligible over the range of this experiment.

V. DISCUSSION OF HEAT LOSS RESULTS

A. Air and Argon Results at Atmospheric Pressure

Sample data for a 1×10^{-4} in. diameter 90Pt-10Rh wire are shown in Figure 3. This heat loss measurement was carried out in air at atmospheric pressure with a wire temperature of about 880°F. In the upper diagram, the data are plotted in the usual coordinates used for hot-wire calibrations. The convex curvature when plotted with the square root of velocity is apparent. From the raw data of this figure, the Nusselt number vs. Reynolds number plot (based on film temperature) of the lower figure was constructed. Least-squares fitting of the data indicated that the proper Reynolds number exponent for these data was 0.34. No end-loss or temperature jump corrections have been applied to the Nusselt number.

To determine the variation of A, B, n with wire temperature, a series of heat loss measurements in air and argon were carried out with a given wire over a range of wire temperatures. In each case, both the corrected and uncorrected Nusselt numbers were computed and their variation with Reynolds number subjected to least-squares fitting. The wire used was 90Pt-10Rh with a diameter of 1×10^{-4} inches. The variation of the quantities A and B as well as the exponent n with $\tau + 1$ are shown in Figure 4 for the case wherein corrections for end-loss and temperature jump have been applied. The Knudsen number for the latter correction was based on wire temperature. Included in these plots are also the results from three heat loss measurements carried out with a different wire (designated wire II) of the same material and diameter. The solid lines represent the least-squares fitting of the logarithm of each quantity vs. $\ln(\tau+1)$. Thus the

values at the abscissa origin are the heat loss "constants" in the limit $\tau \to 0$. To the right of each curve are given the logarithmic slopes determined by the fitting process.

It may be seen that the variation of A with wire temperature is very small and is probably within the experimental accuracy involved in determining its variation. Further, the agreement between the actual numerical value of A for wires 30 and 11 is good. This is an indication of the closeness in $\alpha_0 R_0/\ell$ between the two wires. There is, however, a significant difference in A for air and argon.

The variation of B with temperature has a definite non-zero value; it appears to vary with $\tau+1$ to about the 1/5 power for argon and to the 1/7 power for air. Also the values of B are somewhat higher in argon than in air. While the dependence of B on temperature appears to be about the same for wire 11 as for wire 30, the values of B are slightly higher in the former case. This difference probably results from a difference in actual diameter of the two wires (the nominal value was used in the computations).

With regard to the variation of n with temperature, the variation is small but probably significant. In argon, the logarithmic slope is about 1/10 and in air about 1/8. The numerical values of n agree well between wire 30 and wire 11. In argon, however, an exponent consistently smaller than that in air was found.

B. Results for Small Knudsen Number in Air

Turning to Figure 5, the variation of the quantities A, B, n were here determined from data supplied by heat loss measurements at high ambient density levels. In general, the pressure for each of these measurements varied from about 2 atmospheres to 8 atmospheres.

Thus, although the Knudsen number based on wire diameter and mean free path of the free stream varied during a given test, in each case the pressure level was sufficiently high to render the Knudsen number considerably less than 0.02 and hence the temperature jump correction essentially negligible for the 1×10^{-4} in. diameter wire. Also plotted in Figure 3 are values of A, B, n obtained for a wire of 2.5×10^{-4} in. diameter at atmospheric pressure. Here again the jump correction is small. Nevertheless, in each case temperature jump corrections using a Knudsen number based on wire temperature as well as end loss corrections have been applied.

The variation of A with temperature appears to be somewhat larger than in the previous case; it appears here to vary with about the 1/4 power of wire temperature for the small wire. For the large wire, a similar variation with temperature appears, however, there is a striking difference in the magnitude of A between the small and large wires. No satisfactory explanation for this difference is apparent; it seems to be well above possible errors due to incorrect determination of $\alpha_o R_o / \ell$.

The quantity B varies strongly with temperature, considerably more so than in the case at atmospheric pressure. The variation here with about the 1/3 power of temperature is greater than for any other quantity. It may be noticed that a similar variation seems to occur for the large wire as well and further that the numerical values of B agree well between the two wires. The latter is a good indication that the actual wire diameters were close to the nominal ones used in the computation.

A variation of n with $\tau+1$ very nearly identical to the variation at atmospheric pressure may be seen in the lowest curves

of Figure 5. The agreement between the large wire and the small wire at high density is also quite close.

C. Large Wire at Low Density

The final case studied in air was the heat loss from the large wire at low density. Here, each heat loss measurement was carried out at a fixed free stream density level, lower than atmospheric, and over a range of wire temperatures. It should be pointed out that, because of the limited running time for each heat loss measurement at low density a considerable diminuition of experimental accuracy was inevitable and consequently a larger scatter in the data is to be expected.* Nevertheless, a fair amount of consistency with the previously discussed results was obtained. The variation of A, B, n with wire temperature at a free stream density level of about 0.2 atmospheres is shown in Figure 6. Values corrected for end loss and temperature jump as well as uncorrected values are plotted together. In addition, the quantities from Figure 5 for the large wire at atmospheric pressure are also plotted. A set of heat loss measurements at about 0.5 atmospheres with the large wire was carried out but is not shown; the results will be given later.

Considering first the variation of A with temperature in Figure 6, it is seen that the corrected data at low pressure show a fair agreement with the atmospheric pressure data. To one significant figure (which in view of the scatter is certainly the most that may be considered), the slope A agrees with the value from Figure 5. That the corrections for end loss and, particularly, for

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* The weak dependence of A, B, n on wire temperature calls for extremely accurate and consistent measurements in order to be able to detect this dependence; it is believed that this fact is principally responsible for the divergence of opinion in the literature over the proper value for these quantities, particularly for n.

temperature jump have a significant effect on the slope is evident from a comparison of the corrected and uncorrected values of A. The fact that, when corrections are applied, the values of A agree fairly well with the atmospheric pressure values, is support for at least the approximate validity of the correction procedure.

The variation of B with temperature agrees extremely well with the atmospheric density case for wire 31. It is probable that this close an agreement is fortuitous in view of the experimental scatter. The uncorrected values of B show a slightly different dependence on $\tau+1$ than the corrected values but not such a striking difference as for the quantity A.

Finally, the exponent n seems to vary with temperature in a way somewhat resembling the case for smaller Knudsen number; again the scatter precludes any precise comparison. There does, however, appear to be a discernible difference in the absolute value of n between these results and the case for smaller Knudsen number. The uncorrected values of n also differ considerably from the corrected values, the former giving a larger dependence on wire temperature.

D. Summary of Data

For ease of comparison, a summary of the data discussed above is given in the table below. The logarithmic slopes and the intercepts at $\tau=0$ are given for each case. Included are the results from the large wire at 0.45 atmospheres. In addition, in view of the previous remarks on the argon results, an analysis of the argon data was carried out assuming a temperature jump correction of C = 4 as opposed to the value C = 2 taken for the other measurements. For all cases, values computed without corrections, as well as with, are also given.

Table 2

Wire	Gas	Pressure (atm)	$K_n(T_O)$	Correction C, end loss	$\hat{A}$	$\hat{B}$	$\hat{n}$	A(0)	B(0)	n(0)
30	Air	1	0.02	2, E.L.	-0.04	0.15	-0.13	0.30	0.44	0.515
				0,0	-0.15	0.03	-0.20	0.32	0.45	0.49
	Argon			2, E.L.	0.05	0.21	-0.10	0.23	0.50	0.45
				0,0	-0.21	0.14	-0.23	0.27	0.51	0.44
				4, E.L.	-0.03	0.41	-0.09	0.27	0.48	0.49
	Air	>2	<0.02	2, E.L.	-0.27	0.33	-0.14	0.30	0.44	0.505
				0,0	-0.65	0.38	-0.18	0.31	0.47	0.50
11,30	Air	1	0.02	2, E.L.	-0.04	0.14	-0.13	0.31	0.44	0.51
				0,0	-0.18	0.02	-0.22	0.33	0.45	0.49
31	Air	0.21	0.04	2, E.L.	-0.07	0.36	-0.04	0.58	0.43	0.62
				0,0	-1.1	0.63	0.87	0.87	0.38	0.68
		0.45	0.02	2, E.L.	-0.12	0.39	-0.02	0.58	0.40	0.60
				0,0	-0.60	0.55	-0.43	0.84	0.35	0.68
		1	0.01	2, E.L.	-0.33	0.46	-0.26	0.64	0.43	0.59
				0,0	-0.29	0.18	-0.18	0.72	0.52	0.53

The following additional remarks may be made about these data:

1) The differences in variation of A, B, n with τ+1 between corrected and uncorrected values are in each case considerable and are of course most noticeable when the Knudsen number is appreciable (approximate values of Knudsen number based on free stream properties are given in the table as $K_n(T_O)$).

2) Only two significant figures are given for each quantity since certainly no more accuracy can be attached to the data than this; however, in view of interest in determining the exponent n for the case $\tau \rightarrow 0$, the rounding-off was done to three figures for the case of air at atmospheric pressure (wire 30) and at high pressure. It is seen that the exponent may be taken either as 0.52 or 0.51 in the first case and 0.50 or 0.51 in the second, when rounded to two figures.

3) Use of SC = 4 for the argon data has brought the results for A(0), B(0), n(0) closer to the values obtained for air, although

still not in agreement. From equation (11), an accomodation coefficient of about 0.6 is implied with use of C = 4 as opposed to f = 0.95 for C = 2.

E. Heat Loss in Helium

A striking demonstration of the temperature jump effect is provided by several heat loss measurements carried out in helium. Because of the low density of helium, the wire Knudsen number for the 1×10^{-4} in. diameter wires is of the order of 0.1 at atmospheric pressure. In addition, it seems to be well established that the accomodation coefficient for helium on material such as platinum* is considerably smaller than for air. The combination of these factors results in a large value of the temperature jump correction and consequently, without the correction, a marked decrease in the quantity A is to be expected. Such a decrease was in fact found. Figure 7 gives results obtained for a platinum wire of 1×10^{-4} in. diameter operated at a fairly low value of τ in air, argon and helium. The lower circles are the uncorrected values of Nusselt number in helium, and data on the upper right represent the Nusselt numbers recorded for air and argon. In view of the large correction required for helium and the lack of precise knowledge concerning its accomodation coefficient, the data in air and argon were used to compute (on the basis of equation (11)) the accomodation coefficient for helium and therefore the correction, C. Specifically, at a single Reynolds number, the corrected Nusselt number for air or argon (which was essentially identical for these gases) was compared with the uncorrected value

*Or more precisely, for helium on air on platinum. The accomodation coefficient depends strongly on the surface condition, e.g. see Kennard (1938), p. 320 et seq.

for helium and the correction C required to bring the helium data into coincidence with the other data was computed. Then this value of C was applied to the remainder of the helium data at other Reynolds numbers. The resulting corrected values for helium are plotted as the crosses in Figure 7. For these data, the computed value of C was 5.0 as noted on the figure.

In Figure 8, results from a different wire (of the same nominal diameter) at higher values of τ are plotted. Again, the lower data are for helium uncorrected for temperature jump and the same procedure was applied in computing the value of C required to bring the helium data into coincidence with the air data at a single value of Reynolds number. It is noted that a value of C of 11.7 was obtained; this is considerably in excess of that found at the lower value of τ. It should be mentioned that these values of C were computed assuming a Knudsen number based on gas properties evaluated at the free stream temperature.

The different values for C found at different wire temperatures indicate the strong dependence of accomodation coefficient on wire temperature. In Figure 9 the values of accomodation coefficient computed (by the above-described computation process) at each wire temperature employed are plotted against film temperature. The lower set of data (unflagged) corresponds to values computed by taking a Knudsen number based on free stream temperature, $K_n(T_o)$, and the upper set (flagged) by basing it on wire temperature. The dotted line represents data reported by Hartnett (1961) which were taken by Oliver and Farber for helium on platinum (they were obtained by a conduction heat loss measurement of f). The agreement in trend with wire temperature is good although the absolute values are somewhat different.

F. Conclusions

What conclusions may be drawn concerning the heat loss from cylinders at low Reynolds numbers from the above information? The specific questions in point are:

1) Can the heat loss law be accurately represented by an equation of the form

$$Nu = A + BR^n?$$

2) If so, how do the quantities A, B, n vary with temperature? Does this temperature variation depend on Knudsen number?

3) Can the data at non-negligible Knudsen number be reduced to the case of $K_n \rightarrow 0$ by the simple relation (10)?

With regard to question 1), an affirmative answer is supported by the following evidence: for a given wire diameter (wires 30 and 11) in a given gas (air), nearly identical values were found for A, B, n at $\tau \rightarrow 0$ whether the wire was operated at atmospheric density and the velocity alone varied or whether the wire was operated at density levels up to eight times atmospheric and the density and velocity simultaneously varied In addition, for a wire 2.5 times this diameter, a nearly identical value for B at $\tau \rightarrow 0$ was found over a range of density levels from atmospheric to about 0.2 atmospheres. In contradiction to these conclusions, when the smaller wire was operated in argon, somewhat different values for A, B, n at $\tau \rightarrow 0$ were found; further, a significant difference from the small wire values was noted in the values of A and n for the large wire over all density levels. There are two explanations which may be offered to account for these discrepancies, over and above the experimental scatter:

First, the corrections employed may not be strictly applicable: this premise is related to 3) above and will therefore be discussed later.

Second, the simple heat loss law assumed is, in fact, not precise but merely a rough approximation to the actual law.

It appears that further study of the specific discrepancies noted is required to decide among the alternative explanations given. The second explanation leads directly to the problem of what other parameters are relevant. The discrepancies in the large wire data might be attributed to an aspect-ratio effect (the possibility of which was suggested by Cole and Roshko, 1954); this is discussed later. With regard to the argon data, the only possible additional parameters are Pr and γ but in what way these might enter in the heat loss law is unknown; the difference in Pr between air and argon is only a matter of a few percent at any rate.

Turning to question 2), it is apparent from the results discussed that A, B, n all vary somewhat with wire temperature. In addition, even after corrections have been applied, there is a difference between the variation with temperature at non-negligible Knudsen number and for $K_n \to 0$ at least in the quantities A and B. Thus, while the temperature jump correction may well account for the overall decrease in Nusselt number, it does not apparently take into account the effect of the jump on the temperature distribution about the wire and the consequent difference in dependence of heat transfer on wire temperature level at different Knudsen numbers. The values of A and B appear to decrease in magnitude as the Knudsen number increases both for the small and large wires. However, n seems to be constant for the small wires and diminishes somewhat for the large wire with increasing Knudsen number. There is insufficient data to obtain a meaningful plot of A, B, n vs. Knudsen number.

To properly answer question 3) requires more precise information on the accomodation coefficient. Since the Knudsen numbers in air are all quite small, as are the corresponding corrections, it is difficult to make accurate conclusions on the validity of the temperature-jump correction. In the case of helium, where the Knudsen number is considerably larger, the accomodation coefficient is not adequately known. Nonetheless, the general effect of the correction on the data of wire 31 as shown in Figure 6, as well as the fact that the application of the correction to the helium data appears to produce results consistent with the other heat loss data (and gives a reasonable value for the accomodation coefficient and its variation with temperature), all indicate the general validity of the correction. The improved agreement in the argon data when a smaller accomodation coefficient is assumed may give further support to this view, pending the determination of accurate values for f.

G. Comparison with Results of Other Investigators

The heat loss quantities found will first be compared with previous results for the case $\tau \to 0$. Two typical theoretical solutions will be considered: the approximate result given by King

15) $$Nu = \frac{2}{\ln \frac{8}{RPr} - 0.67E} \qquad RPr < 0.08$$

16) $$Nu = \frac{1}{\pi} + \sqrt{\frac{2}{\pi} RPr} \qquad RPR > 0.08$$

and the Oseen solution by Cole and Roshko,

17) $$Nu = \frac{2}{\ln \frac{8}{RPr} - E} \qquad R \to 0$$

In these equations E is Euler's constant.

Of the available experimental results, the most precise appear to be those of Collis-Williams. They give

18) $$Nu = 0.24 + 0.56\ R^{0.45} \qquad 0.02 < R < 44$$

19) $$Nu = \frac{2.09}{\ln 11.4/R} \qquad 0.02 < R < 0.5$$

The often quoted correlation of McAdams (1954) of a wide variety of data from various investigators is

20) $$Nu = 0.32 + 0.43\ R^{0.52} \qquad 0.1 < R < 1000$$

The result obtained in the present work for wire 30 at atmospheric pressure in air (with corrections based on $K_n(T_w)$ applied) is

21) $$Nu = 0.30 + 0.44\ R^{0.52} \qquad 0.4 < R < 4$$

where the exponent has arbitrarily been rounded off from 0.515 to 0.52. In argon,

22) $$Nu = 0.23 + 0.50\ R^{0.45} \qquad 0.4 < R < 4$$

For purposes of comparison, a Prandtl number of 0.70 will be taken in 15) to 17). Each of the equations 15) to 22) is plotted in Figure 10 over the limits specified. With the exception of the

result of King for large Reynolds number, there appears to be only small differences among the various results. As pointed out by Collis and Williams, their data approach the theoretical result of Roshko and Cole at small Reynolds number. At higher Reynolds number, the Collis-Williams data appear to be somewhat above the results of the present work which seems to agree closely with McAdams' correlation. There is only a very small difference between the argon and air curves in the present work.

It has been mentioned previously that Cole and Roshko have suggested a possible dependence on aspect ratio of the heat loss from finite cylinders. For pure conduction from an ellipsoid they point out that

$$\text{23)} \qquad Nu = \frac{2}{\ell n 2\, R} \qquad (R = 0)$$

for large aspect ratio. Taking $A\!\!\!/R = 800$ which corresponds to the value for wire 30 then from 23)

$$Nu = 0.27 \qquad (R = 0)$$

and this value is given on the ordinate axis in Figure 10. It should be noted that this value is very close to the value of A obtained for wire 30. This fact, combined with the finding that the variation of A with temperature is nearly zero (at least for the corrected data obtained at atmospheric pressure), is strong indication that the quantity A in the assumed heat loss law represents pure conduction heat loss from the cylinder. If so, then it may well be aspect-ratio dependent in a way quite analogous to 23). The somewhat different variation of A with temperature in the high density tests remains to be explained, however.

The aspect-ratio of wire 31 was about 300 hence the pure conduction Nusselt number is 0.31, a value considerably less than the value of 0.64 obtained for wire 31 at atmospheric density. It does not then appear possible to explain the discrepancy in A between wires 31 and 30 on the basis of different aspect-ratio.

With regard to the heat loss law for finite τ, a wide divergence in its dependence on τ appears in the literature. From a consideration of many factors, the results of Collis-Williams again appear to be the most precise. Unlike the work carried out here, however, these investigators grouped the results at all wire temperatures together to determine a single Reynolds number exponent, independent of wire temperature. For this reason, it is difficult to compare their results with the present results. It is significant, however, that the exponent found by them was 0.45, a value less than that which has been reported previously in the literature. The work here indicates that, in general, the exponent diminishes with wire temperature from a value near 0.5 at $\tau \to 0$ to lower values at large temperatures. Consequently, grouping together results taken over a range of wire temperatures would naturally tend to yield an exponent lower than 0.5 as the best data fit (this fitting was done by "trial and error" in the work of Collis and Williams). It is believed then that, for the assumed heat loss law given by equation (1), values of the exponent n lower than 0.5 (as found by Collis-Williams) result from the effect of temperature loading; when $\tau \to 0$, the exponent is very nearly 0.5, which is the conventionally assumed value.

VI. VELOCITY AND TEMPERATURE FLUCTUATION SENSITIVITY

A. Preliminary Remarks

Once the steady heat loss law for a hot-wire is known, the sensitivity of the wire to fluctuations in the stream passing over it may be obtained by perturbation of the mean flow. A linearized sensitivity equation thereby results for the (small) fluctuation quantities in which each such quantity is weighted by a sensitivity coefficient; these coefficients provide a measure of the contribution of the quantity in question to the total AC signal output of the heated wire. A general derivation of the sensitivity of the hot-wire to fluctuations in the stream velocity and temperature will first be given here; the details of the transient response including lag effects will be ignored*. Subsequently, simplified forms of the velocity sensitivity for special cases will be deduced and the errors introduced by their use discussed. Finally, the effects of end loss to the wire supports will be considered. Although they are not usually important in hot-wire anemometry, for purposes of estimation the effects of radiation and a non-linear resistance-temperature wire behavior will be included; an arbitrary reference temperature upon which to base the fluid properties is employed.

B. Sensitivity Equation

The heat loss law found empirically for a long heated cylinder placed normal to an incompressible stream at low Reynolds number and negligible Knudsen number is given by (1). The wire resistance is taken as

$$R_w = R_o \left[1 + \alpha_o (T_w - T_o) + \beta_o (T_w - T_o)^2 \right]$$

---

* For a discussion of this response, see Corrsin (1961).

consequently

25) $$\tau = \frac{a_w}{\alpha_o}(1-\delta)$$

to order δ, where

$$\delta = \frac{\beta_o}{\alpha_o^2} a_w \quad .$$

Using a film temperature

$$T_f = T_o + t\,(T_w - T_o)$$

where t is an arbitrary constant, the fluid properties evaluated at this temperature are taken to be

26) $$k = k_o\,(\,1 + 2\,tha_w)$$

27) $$\nu = \nu_o\,(\,1 + 2\,th_1 a_w)\,(1 + 2\,th_2 a_w)$$

where

$$h = \frac{(dk/dT)_m}{2\alpha_o k_o}(1-\delta),\ h_1 = \frac{(d\mu/dT)_m}{2\alpha_o \mu_o}\ (1-\delta),\ h_2 = \frac{1-\delta}{2\alpha_o T_o} \quad .$$

In the expression (1), a functional dependence

$$A = A(\tau)$$

$$B = B(\tau)$$

$$n = n(\tau)$$

is assumed. The quantity A is taken to include the heat loss due to radiation so that a separation

$$A(\tau) = A_R(\tau) + A_f(\tau) \tag{28}$$

is effected where A_R represents heat loss due to radiation and A_f the value of A in the absence of radiation. For the radiation heat loss, the Stefan-Boltzmann law is applied and the wire assumed to radiate to a large enclosure which is at the ambient stream temperature. Consequently,

$$A_R = \frac{d\sigma e}{k} \frac{T_w{}^4 - T_o{}^4}{T_w - T_o} \tag{29}$$

where σ is the Stefan-Boltzmann constant and e the wire emissivity.

The fluctuation quantities of the field relative to their mean value are denoted by $\Delta U/U$ and $\Delta T_o/T_o$; these lead to a fluctuating output voltage of the DC heated wire, $\Delta E/E$. This voltage may be expressed as

$$\frac{\Delta E}{E} = \frac{\Delta R_w}{R_w} \left(1 - \frac{I R_w}{V}\right) \tag{30}$$

where

$$V = -I^2 \frac{\Delta R_w}{\Delta I} \tag{31}$$

and is determined from the characteristics of the circuit into which the wire is placed. In the usual constant-current hot-wire circuit where the wire forms part of a bridge which is placed in series with a large resistor and heated from a battery,

$$V \cong E_b \quad ,$$

the battery voltage.

To obtain the fluctuation sensitivity equation, the expressions (24) to (29) are inserted into (1) and small perturbations of U, T_o, I, R_w are assumed. Linearizing with respect to these quantities and using (30) then gives an expression of the form

$$32) \qquad S_1 \frac{\Delta U}{U} + S_2 \frac{\Delta T_o}{T_o} = S_3 \frac{\Delta E}{E}$$

where S_1, S_2 are then the velocity and temperature fluctuation sensitivities.

Denote by a ^ the logarithmic derivative of a quantity with respect to τ+1; thus e.g.,

$$\hat{A} = \frac{d\ell nA}{d\ell n\ (\tau+1)} \quad ;$$

also call

$$\tilde{t} = \frac{1-t}{t} \quad .$$

The velocity sensitivity is then given by

$$33) \qquad S_1 = -a_w n(1-\frac{A}{Nu})$$

and for the temperature sensitivity,

$$34) \qquad S_2 = \frac{1-\delta}{2h_2} (1-\tilde{t}f_o)$$

where

$$f_o = \frac{2tha_w}{1+2tha_w} \left[\frac{A}{Nu} r_o + \epsilon_o \left(1-\frac{A}{Nu}\right) \right] .$$

In this expression,

$$r_o = \frac{1}{p}\left(1-\frac{\omega_o}{\tilde{t}}\right)\left[1 + s_1 \frac{(1+\tau)^4 - (1+4\tau)}{\tau^2}\right]$$

$$\epsilon_o = 1 - \frac{n}{2t\tilde{t}} \; \frac{1+2tha_w}{1+h_2a_w} \; \frac{h_2}{h} \; g$$

whereIn

$$p = 1 + s_o \; \frac{(1+\tau)^4-1}{a_w(1+2tha_w)}$$

$$\omega_o = \frac{h_2}{th} \; \frac{1+2tha_w}{1+2h_2a_w} \; \hat{A}_f \; \left[1 + \frac{p}{\hat{A}_f} \; \frac{\tau+1}{\tau} \; \delta\right]$$

$$g = 1 + 2t\tilde{t} \; \frac{1+h_2a_w}{1+2th_1a_w} \; \frac{h_1}{h_2} + \frac{w}{n} \; \frac{\tau+2}{\tau+1} \; \left[1 - \frac{\tau+1}{h\tau} \; \delta\right]$$

and

$$s_o = \frac{d\sigma e T_o^3}{A_f k_o(1-\delta)}$$

$$s_1 = \frac{1-\delta}{\omega_o+\tilde{t}} \; \frac{h_2}{th} \; s_o$$

$$w = \hat{B} + \hat{n} \; \ell n \; \frac{pA_f}{B} \; \left(\frac{Nu}{A} - 1\right)$$

The quantity S_3 is given by

35) $$S_3 = \frac{1 + 2a_w\frac{IR_w}{V} + (1+a_w) \; f_1}{1 - \frac{IR_w}{V}}$$

where
$$f = \frac{2tha_w}{1+2tha_w} \frac{A}{Nu} r_1 + \epsilon_1 (1 - \frac{A}{Nu})$$

$$r_1 = \frac{1}{p} (1+\omega_1) \left[1+s_2 \frac{1-(1+\tau)^3 (1-3\tau)}{\tau^2} \right]$$

$$\epsilon_1 = (1-\delta) \epsilon_o$$

$$\omega_1 = \omega_o (1- \frac{1+\omega_o}{\omega_o} \delta)$$

$$s_2 = (1-\delta) \frac{1+\omega_o}{1-\omega_1} s_1$$

C. Simplifications for Velocity Sensitivity

It is clear from these results that a considerable amount of information is required to compute the sensitivities in general. An alternate method of determining the fluctuation quantities in the field is the so-called "mode-diagram" method of Morkovin (1956). In this method, a form of the heat loss law is assumed similar to (1) but with n = ½. Then instead of using analytical expressions as given here, a graphical solution technique is developed. The data required for this solution are a number of values of the heat loss and AC wire output signals at different wire temperatures; having this information then permits all of the fluctuation quantities to be computed. The simplicity of the mode-diagram method rests upon the assumption of a fixed exponent of ½ for the Reynolds number in the heat loss law. However, in the work here, the exponent was found to be a function of wire temperature. Another objection to the graphical solution method is that a large number of wire output signals must be

obtained; since for each of these, a separate square-wave compensation adjustment must be made each of which entails several bridge balancing operations, the method is time-consuming and is not practical when running time is limited. Further, the method precludes obtaining information on the wire heat loss under various flow conditions which is useful in checking that the wire is behaving properly.

With use of the constant-current fluctuation measurement technique, a direct calibration procedure in which the current is held fixed and the wire temperature vs. velocity characteristic is obtained might also be employed and is, in fact, the most logical method. However, the form (1) indicates that considerable difficulty would be encountered in empirically fitting data obtained in this fashion.

When velocity fluctuations are of prime interest, a direct analytical computation employing the general information previously obtained for the heat loss law may be applied when the wire is operated so as to cause it to be principally sensitive to velocity fluctuations. The following simplifications may then usually be made:

1) No free stream temperature variations, $\frac{\Delta T_o}{T_o} = 0$.

2) Effects of radiation negligible; this amounts to taking zero emissivity hence $s_o = s_1 = s_2 = 0$.

3) Linear resistance-temperature behavior, $\delta = 0$.

With these assumptions, and taking $t = \frac{1}{2}$, the quantity f_1 in S_3 becomes

$$f_1 = \frac{ha_w}{1+ha_w} \frac{A}{Nu} \left[(1+\omega_1) + \epsilon_1 \left(1-\frac{A}{Nu}\right) \right]$$

where now

$$\omega_1 = \frac{2h_2}{h} \frac{1+ha_w}{1+h_2 a_w} \hat{A}$$

$$\epsilon_1 = 1-n \frac{h_2}{h} \frac{1+ha_w}{1+h_2 a_w} g$$

$$g = 1 + \frac{h_1}{h_2} \frac{1+h_2 a_w}{1+h_1 a_w} - \frac{2w}{n} \frac{1+h_2 a_w}{1+2h_2 a_w}$$

$$w = \hat{B} + \hat{n} \, \ell n \frac{A}{B} \left[\frac{Nu}{A} - 1 \right]$$

and S_1 remains the same. The sensitivity equation then is

$$\frac{\Delta U}{U} = \frac{S_3}{S_1} \frac{\Delta E}{E}$$

and to compute the velocity fluctuation from the wire output signal requires a knowledge of:

a) the fluid properties k_o, $(dk/dT)_m$, μ_o, $(d\mu/dT)_m$

b) the operating Nusselt number, Nu, and overheat ratio, a_w

c) the heat loss "constants" A, B, n at the wire temperature employed and their variation with temperature, $\hat{A}$, $\hat{B}$, $\hat{n}$

d) the coefficient, α_o, and stream temperature, T_o

e) the voltage across the wire, IR_w, and the supply voltage, V.

The degree of approximation made in assumptions 1) to 3) will be estimated.

1) Assumption of $\Delta T_o/T_o = 0$:

To estimate the effect of temperature fluctuations, the assumptions 2) and 3) are made (for simplicity) so that S_2 becomes

$$S_2 = \frac{1}{2h_2} (1-f_o) \quad ;$$

hence

$$\frac{S_2}{S_1} = -\frac{1}{\tau} \quad \frac{1-f_o}{n(1-A/Nu)} \quad .$$

The ratio S_2/S_1 provides a measure of the relative contribution to the total voltage signal of the temperature and velocity fluctuations. It is clear that within the range of allowable wire temperatures, the wire is principally sensitive to temperature fluctuations. Inasmuch as it is not possible to reduce the relative temperature sensitivity to zero, the contribution of small stream temperature fluctuations to the wire output signal must be assessed. For an isotropic field, the correlation of the (scalar) temperature with the velocity field is zero; the less restrictive assumption of axial symmetry, however, allows a non-zero correlation but since no rational choice is available for the correlation in this or in a more general field, the field will be assumed to be isotropic for the purpose of the present estimates. Then, the ratio of actual rms velocity fluctuation to that computed by assuming no temperature fluctuation may be shown to be, for small temperature fluctuations,

$$\frac{U'_{act}}{U'_{comp}} = 1 - \frac{1}{2} \left(\frac{S_2}{S_1}\right)^2 \left(\frac{T'_o/T_o}{U'/U}\right)^2$$

To obtain an estimate of orders of magnitude, assume

36) $$k = \left(\frac{k_o}{T_o}\right) T_f \ , \quad \mu = \left(\frac{\mu_o}{T_o}\right) T_f$$

and take the typical values $A/Nu = \frac{1}{2}$, $n = \frac{1}{2}$; then

$$\frac{S_2}{S_1} = - \quad \frac{2}{\tau} \frac{4+\tau}{2+\tau}$$

Examples of the errors introduced with small and large temperature fluctuations (about 0.01 and 0.1°C. in absolute value) when the wire is operated at small and large values of τ for a range of relative rms velocity fluctuations are given in the following table:

Table 3

τ	T_0'/T_0 (%)	U'/U (%)	Error (%)
0.1	0.003	0.01	>100
		0.1	70
		1.0	0.7
1	0.03	0.01	>100
		0.1	50
		1.0	0.5

In conventional flow streams produced in the laboratory, a level of temperature fluctuations in excess of 0.01°C. may easily occur unless very special precautions are taken. From the above table, it is then clear that, unless high wire temperatures are employed, the measurement of small velocity fluctuations are at best only approximate and may be grossly in error. Even when the wire is operated at 600°K., the error in measuring velocity fluctuations of the order of 0.1% is about 50% when temperature fluctuations of only 0.01°C. occur. Consequently, the accuracy of measurements reported in the literature of such small velocity fluctuation levels is questionable if some indication that the temperature fluctuation level is negligible is not given. A simple way to provide such an indication experimentally is to operate the wire at more than one (high) temperature level and compute the turbulence level from each measurement, assuming no temperature fluctuation. If the resulting values agree closely, then the measurements are probably satisfactory.

Since it is necessary to operate at high wire temperature to minimize the contribution of possible stream temperature fluctuations to the wire output signal, a knowledge of the wire heat loss law for

such elevated temperatures is mandatory. That is, terms due to the quantities $\hat{A}$, $\hat{B}$, $\hat{n}$ become more important as τ increases. As an example, consider the contribution of $\hat{A}$ alone. Making the assumptions of (36), and again taking $n = \frac{1}{2}$, $Nu/A = \frac{1}{2}$, it is a simple matter to show that velocity fluctuations computed by including A relative to the values computed assuming A is zero are, for small τ, equal to

$$1 + \frac{\tau}{2} \hat{A}$$

and consequently the error committed in ignoring $\hat{A}$ increases directly with τ. Since $\hat{A}$ may typically have a magnitude of 0.2 then at $\tau = \frac{1}{2}$ the error is of the order of 5% and will be larger for higher values of τ. Measurements reported in the literature taken for high values of τ may, therefore, also be considerably in error if the temperature dependence of the quantities A, B, n is not determined. It should be noted further that since the computation of the turbulence energy level involves the square of the velocity fluctuation, the errors committed are then approximately doubled.

2) Assumption of negligible radiation

The effect of wire radiation is determined by the magnitude of the quantities s_o, s_1, s_2. Taking $t = \frac{1}{2}$, $\hat{A} = 0$, $\delta = 0$, and using (36) gives

$$s_2 = s_1 = 2s_o$$

where

$$s_o = \frac{d\sigma e T_o^{\,3}}{A_f k_o} \quad .$$

Using a liberal value of 0.3 for the emissivity and typical values of 0.25 for A_f, 300°K. for T_o, gives for wires of 2.5×10^{-4} inches diameter in air a value of s_o of about 5×10^{-4}. Thus, the contribution of the radiation terms is less than 1% for the maximum values of τ which may be employed and these terms may then safely be omitted even for wires of relatively large diameter.

3) Assumption of $\delta = 0$

The difficulty in precisely measuring the second coefficient β_o makes the estimation of δ uncertain. Spangenberg (1955) has given several values of β_o for 90Pt-10Rh wires which differ considerably. The maximum value of β_o/α_o^2 in the data given is -0.106 and the minimum value is zero. A simple average of these, say -0.05 might be a fair estimate in general. Taking this value, then for $a_w = 0.5$, δ becomes 0.025 hence an error of about 2.5% may be incurred by taking $\delta = 0$. It should be pointed out, however, that if $\delta = 0$ is also assumed in the heat loss data reduction then the use of empirically determined quantities A, B, n will partially counteract errors in the sensitivity expression which results from taking $\delta = 0$. The ultimate test of course is whether the same fluctuations are computed when the wire is operated at different temperatures.

D. Effect of Wire End Losses

1. On computing fluctuation level:

A consideration of the effect of heat loss to the wire supports on the measurement of fluctuation quantities has been given by Betchov (1952). The results there are given in terms of the (small) parameter S (see equation (9)).

The theory of Betchov was carried out assuming a heat loss law of the form

$$Nu = A(\tau) + B \sqrt{R}$$

where Nu and R are based on stream temperature, A is a function of wire temperature and B a constant. It is therefore an extension of earlier theories in which A is taken as a constant, although it does not account for possible variations in B and n with wire temperature. It may be shown, however, that to order S the effect of end loss and of variable A are separable and that consequently end loss corrections may be applied independent of the effect of variable A. It is therefore reasonable to suppose that the same situation will hold with n and B also variable, especially since A is the principal variable of the three.

Expanding the result of Betchov in powers of S gives for the end-loss correction to the fluctuation measurement,

$$K_{EL} = 1 + \frac{1}{2} \frac{1-a_w}{\sqrt{1+a_w}} S ,$$

ignoring terms of order S^2 and smaller. For S of about 0.05, then at $a_w = 0.5$ the correction is only of the order of 1%. This estimate then indicates that the correction may usually be safely ignored. It should be noted that it is assumed in the above that the quantities A, B, n which are used are those for the actual wire heat loss, and are not values corrected for end loss or temperature jump.

2. On time constant

The question of the effect of end-loss on the time constant of the wire was also considered by Betchov. It was found that the

wire frequency characteristic is approximately the same with or without end-loss and the required correction to the time constant was computed (however, see Corrsin, 1961, for a discussion of the applicability of this computation). Again, it may be shown from these results that the effects of variable A and end-loss can be separated with the result that the relation between the time constant with and without end-loss is, to order S,

$$\frac{M}{M^+} = 1 - \sqrt{1+a_w}\ \frac{S}{2}$$

which indicates an expected deviation of only a few percent. Since the time constant is usually determined experimentally by the square-wave method, it is then necessary to know the effect of end-loss on the current-fluctuation time constant. To order S, the current time constant is identical with or without end-loss, consequently, the value determined for the velocity will be in error by the (small) amount given by the above expression for the case considered by Betchov.

E. Variation of Time Constant with Wire Temperature

Without the end-loss effect, the result for the time constant given by Betchov may be expanded to give

$$38) \qquad \frac{M}{M_o} = 1 - G + \frac{2+a_w}{1+a_w} G^2$$

ignoring terms of order G^3 and smaller. In this expression, M is the time constant for $A = A(\tau)$, M_o the time constant for constant A, and the parameter G, which is a measure of the effect of variable A, is

$$G = h_2 a_w \, (1+a_w) \, \frac{A}{Nu} \quad ;$$

also

$$M_o = \frac{c_w S_w d^2}{4k} \left(\frac{1+a_w}{Nu} \right)$$

Equation (38) is plotted in Figure 11 for two values of a_w. The experimental points were taken at random from time constant values obtained by the square wave method in the course of other work performed here which is to be reported on later. To eliminate the effect of uncertainties in the wire diameter and other physical properties of the wire, a normalizing procedure was used in plotting these data. For a given wire, the value of M/M_o recorded for the lowest value of G was divided into the rest of the experimental values of M/M_o. Then, since this value of G was not actually zero, a correction was applied by finding the theoretical value of M/M_o at this value of G and correcting the remainder of the ratios by multiplying by the theoretical value there. Although there is some scatter in the data, the general shape of the theoretical prediction appears to be verified. It should be noted that since only terms up to order G^2 were included, the theoretical curve plotted may be in error by several percent for G = 0.3 or greater.

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12. Abstract: An experimental study of the forced convection heat loss from long thin cylinders placed normal to an incompressible flow has been carried out. Tests were made in three different gases (air, argon, helium) at atmospheric pressure and in air over a pressure range 0.2 to 8 atm. Particular attention was given to determining the effect of cylinder temperature on the heat loss. It was found that the "constants" employed in the usual empirical representation of this heat loss vary with cylinder temperature in a way dependent on Knudsen number, results in helium giving a marked indication of this. A velocity fluctuation sensitivity equation was derived which includes the effects of this temperature variation.

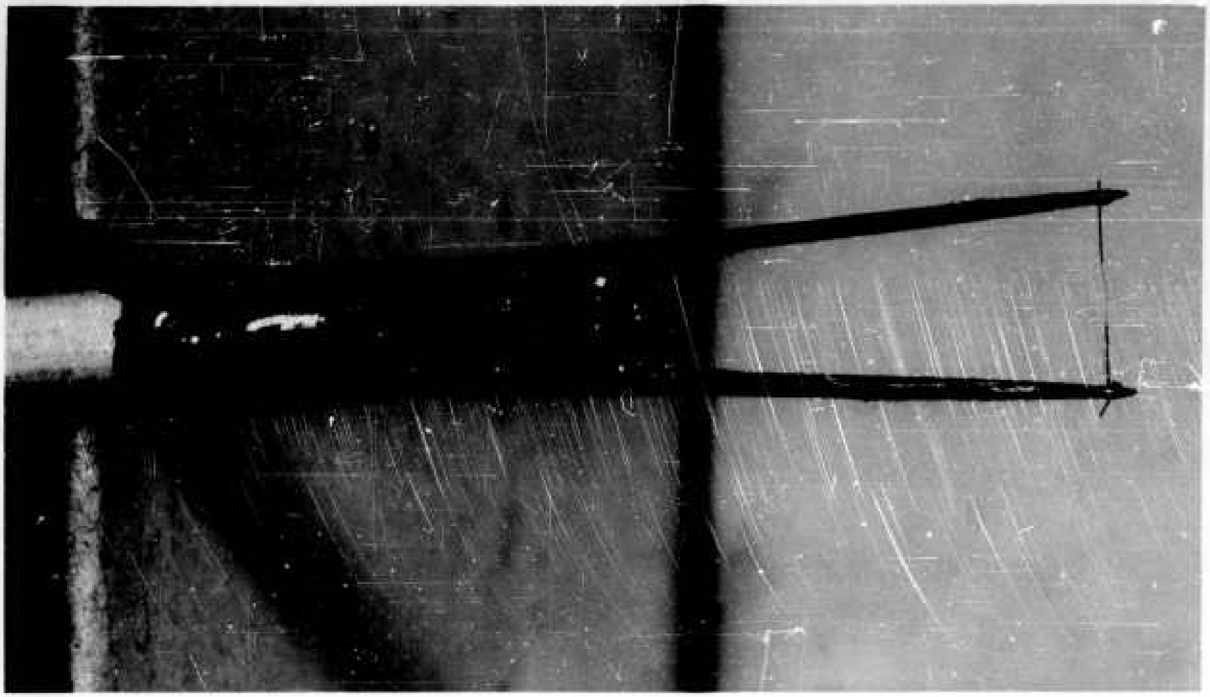

Figure 1. Hot wire probe tip

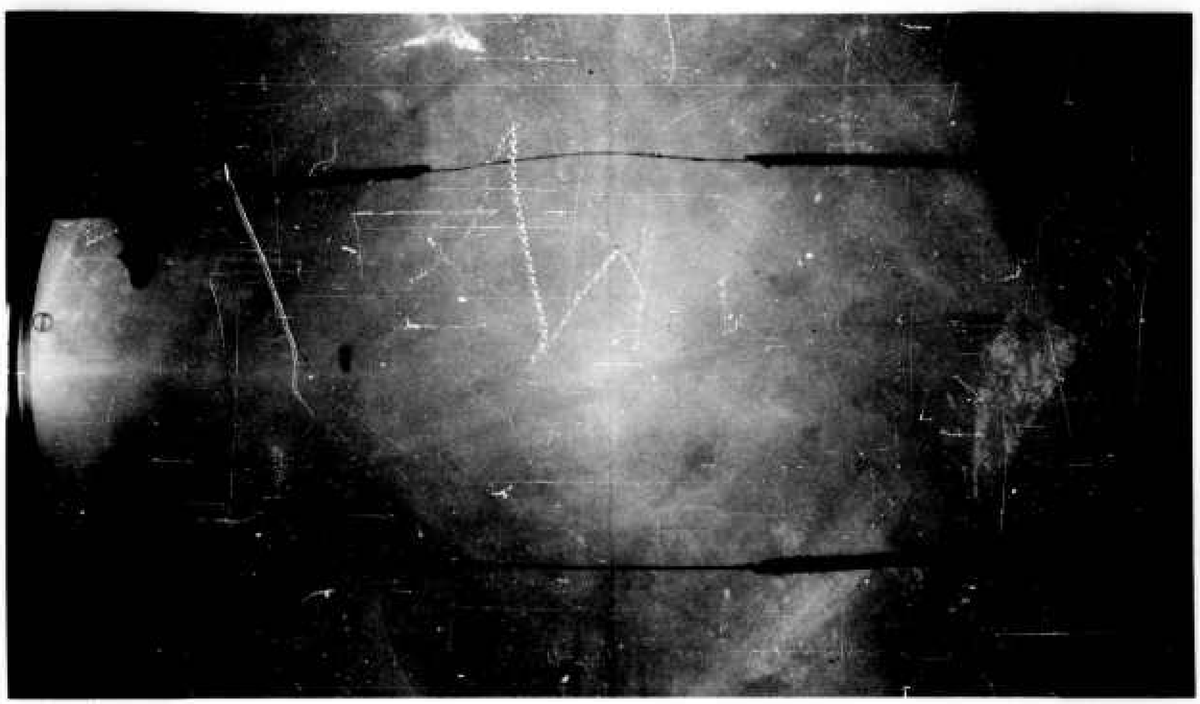

Figure 2. Actual wires used. Optical comparator photograph, upper wire is 1×10^{-4} inches diameter (wire 30), lower wire is 2.5×10^{-4} inches diameter (wire 31)

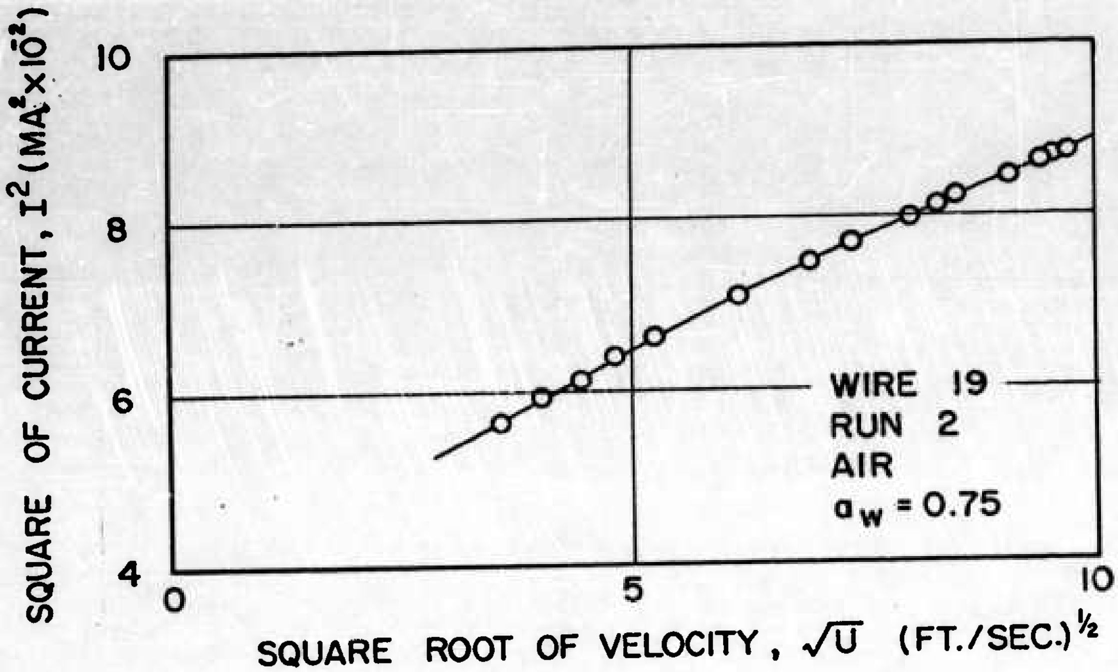

FIG. 3 A - USUAL DIMENSIONAL HOT WIRE CALIBRATION.

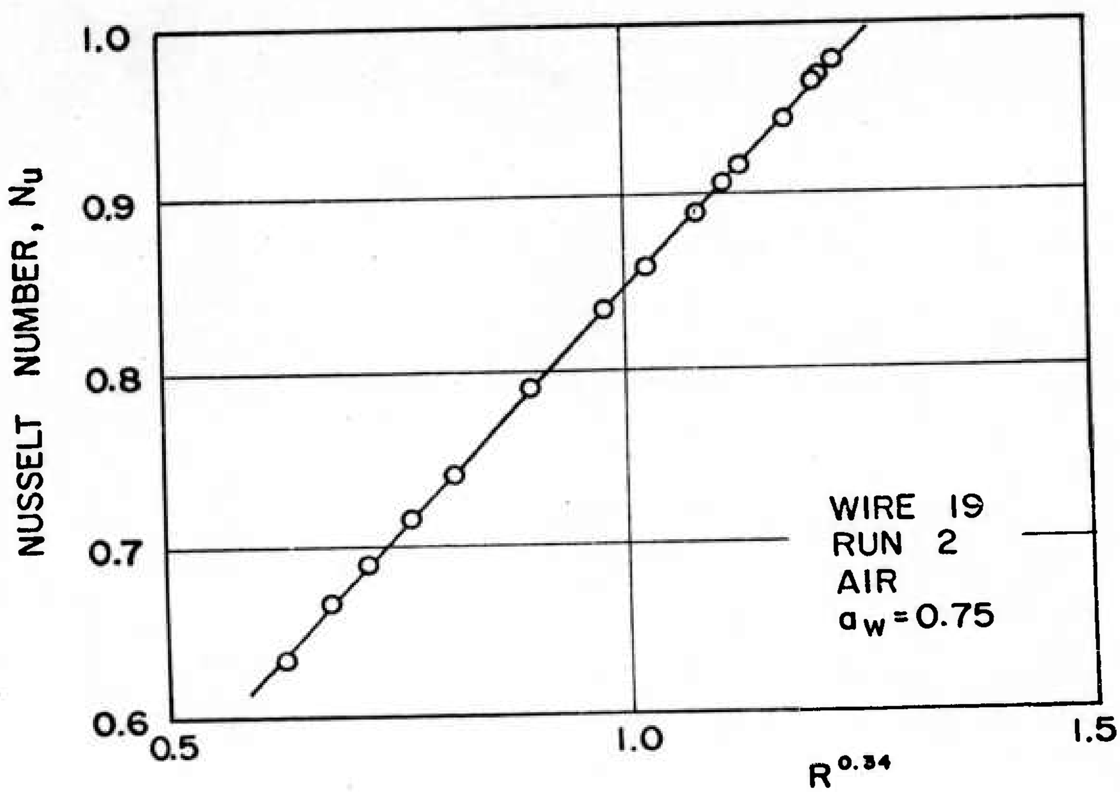

FIG. 3 B - DIMENSIONLESS HEAT LOSS FOR ABOVE DATA.

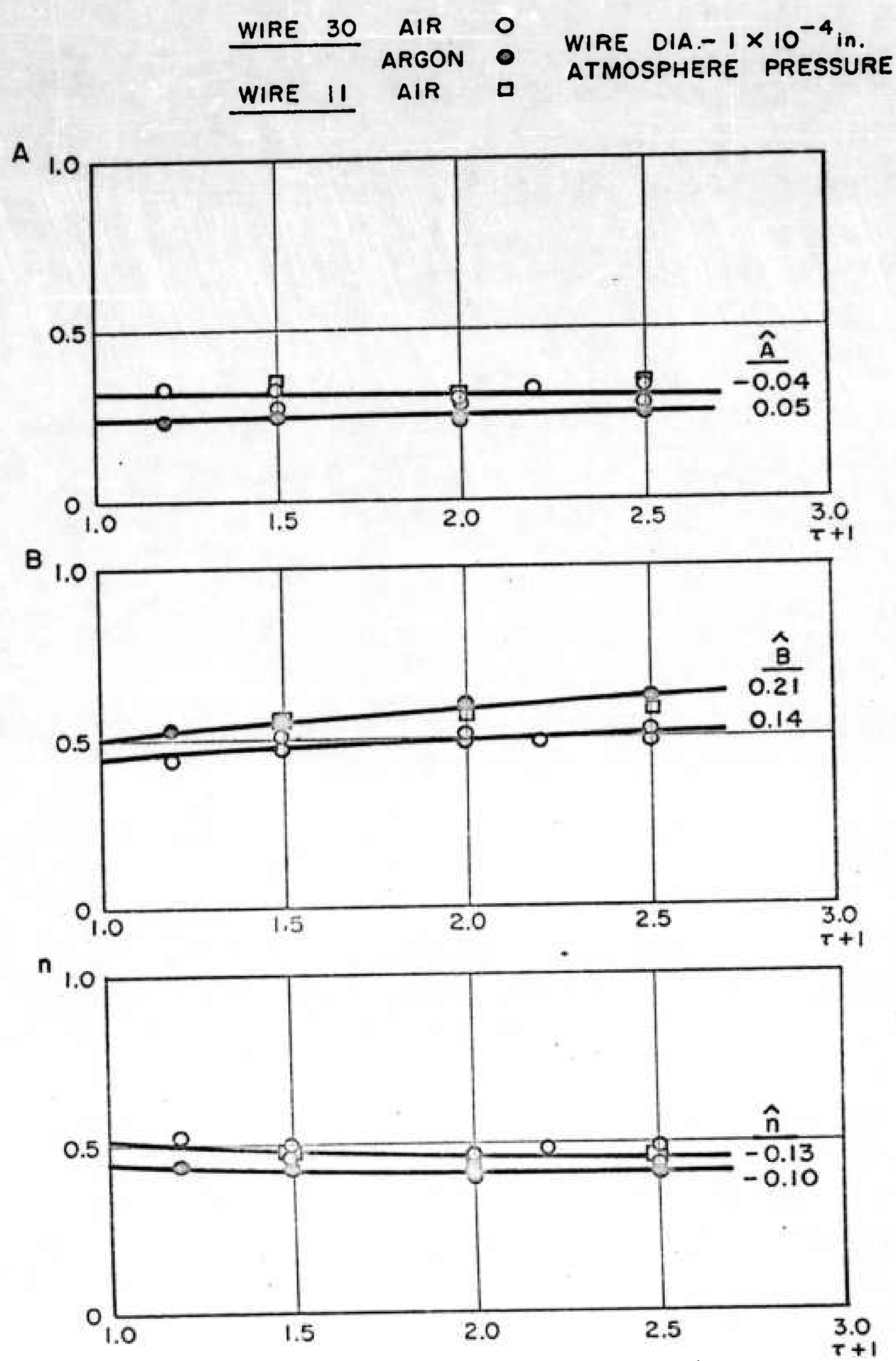

FIG. 4 VARIATION OF A, B, n, WITH $\tau + 1$ - SMALL WIRES, ATM. PRESSURE.

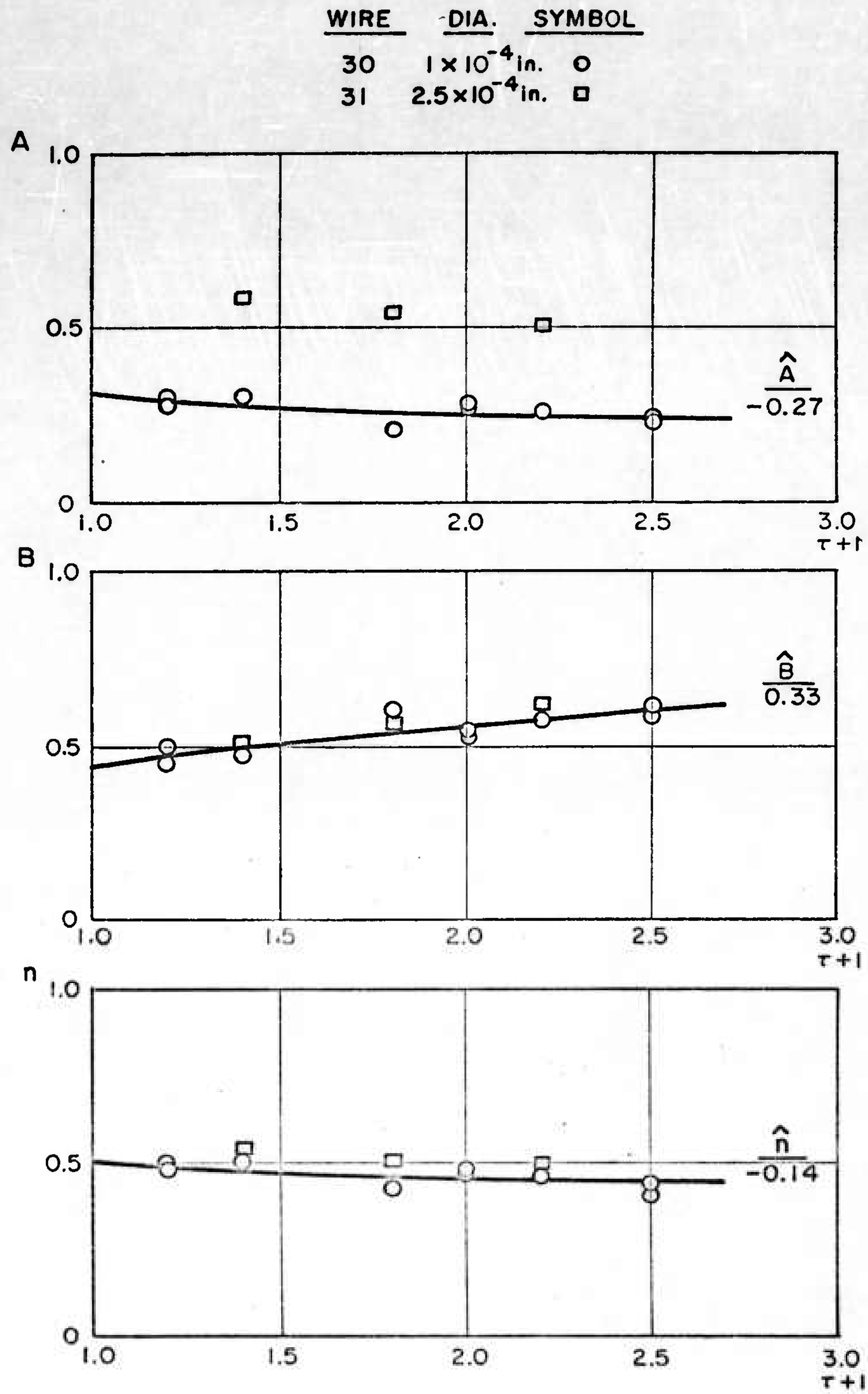

FIG. 5 VARIATION OF A, B, n, WITH $\tau + 1$ -
Kn → 0

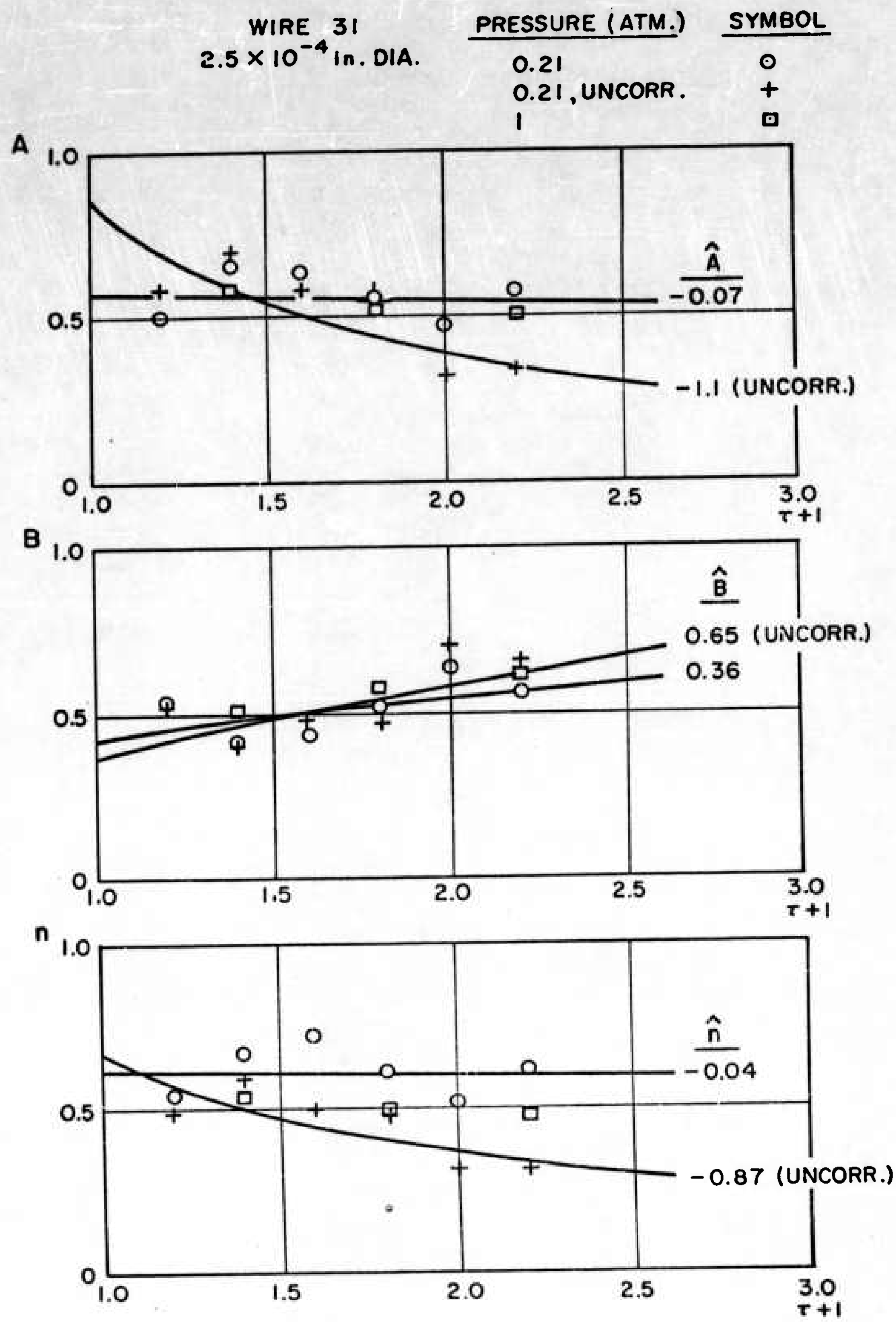

FIG. 6 VARIATION OF A, B, n, WITH $\tau + 1$
WIRE 31

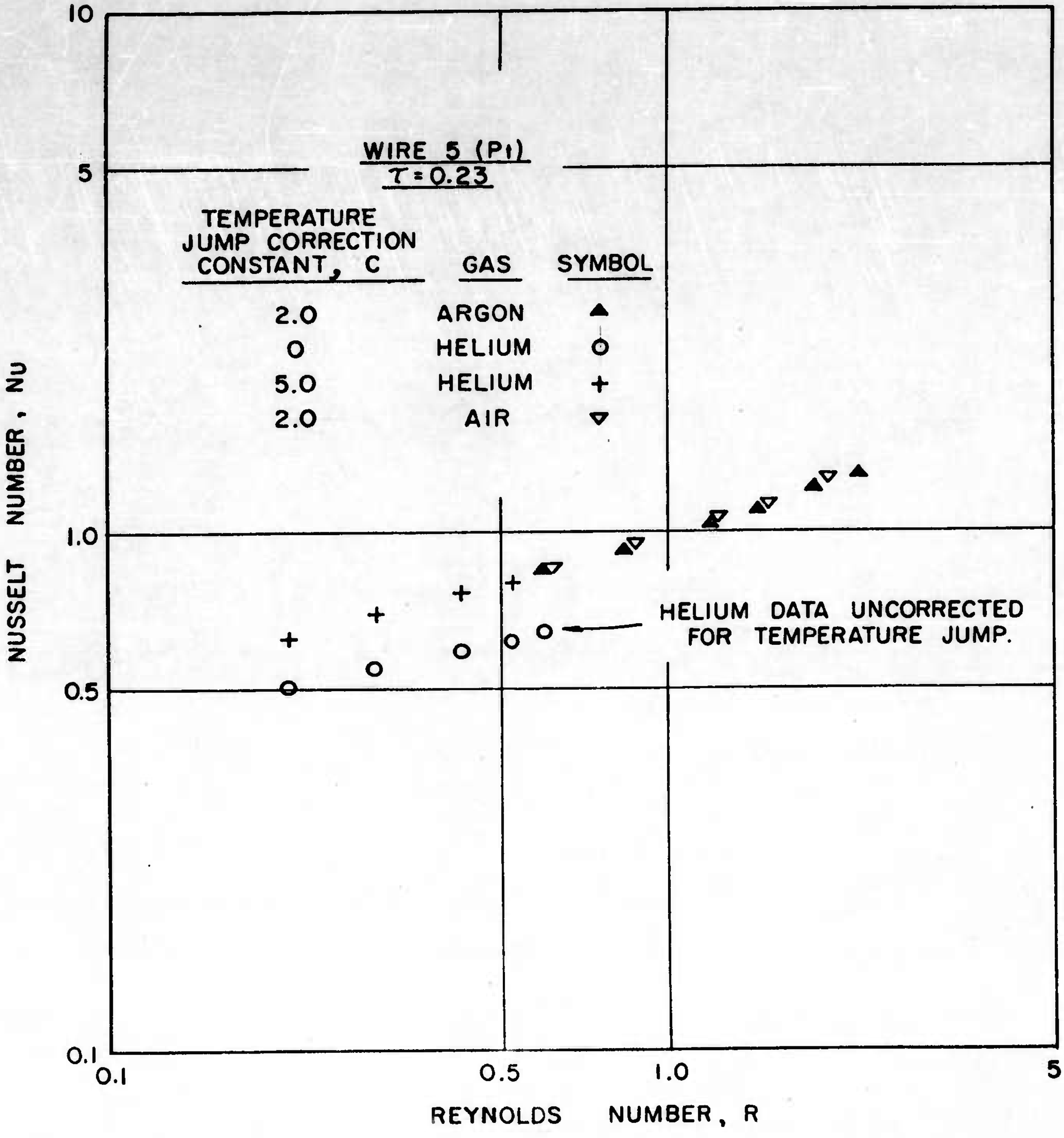

FIG. 7 - Nu vs. R FOR AIR, ARGON, HELIUM - WIRE 5

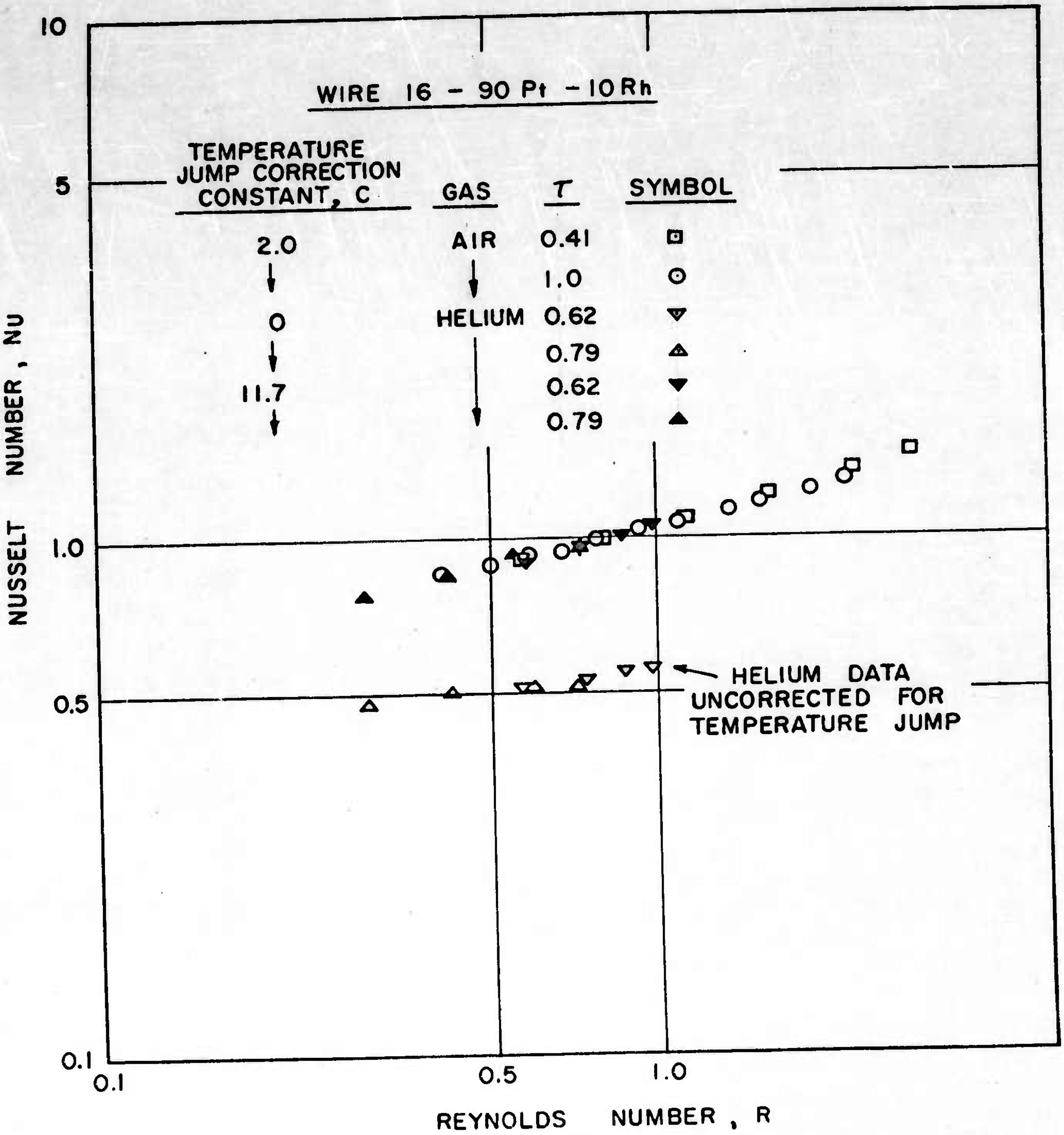

FIG. 8 - Nu vs. R FOR AIR AND HELIUM - WIRE 16

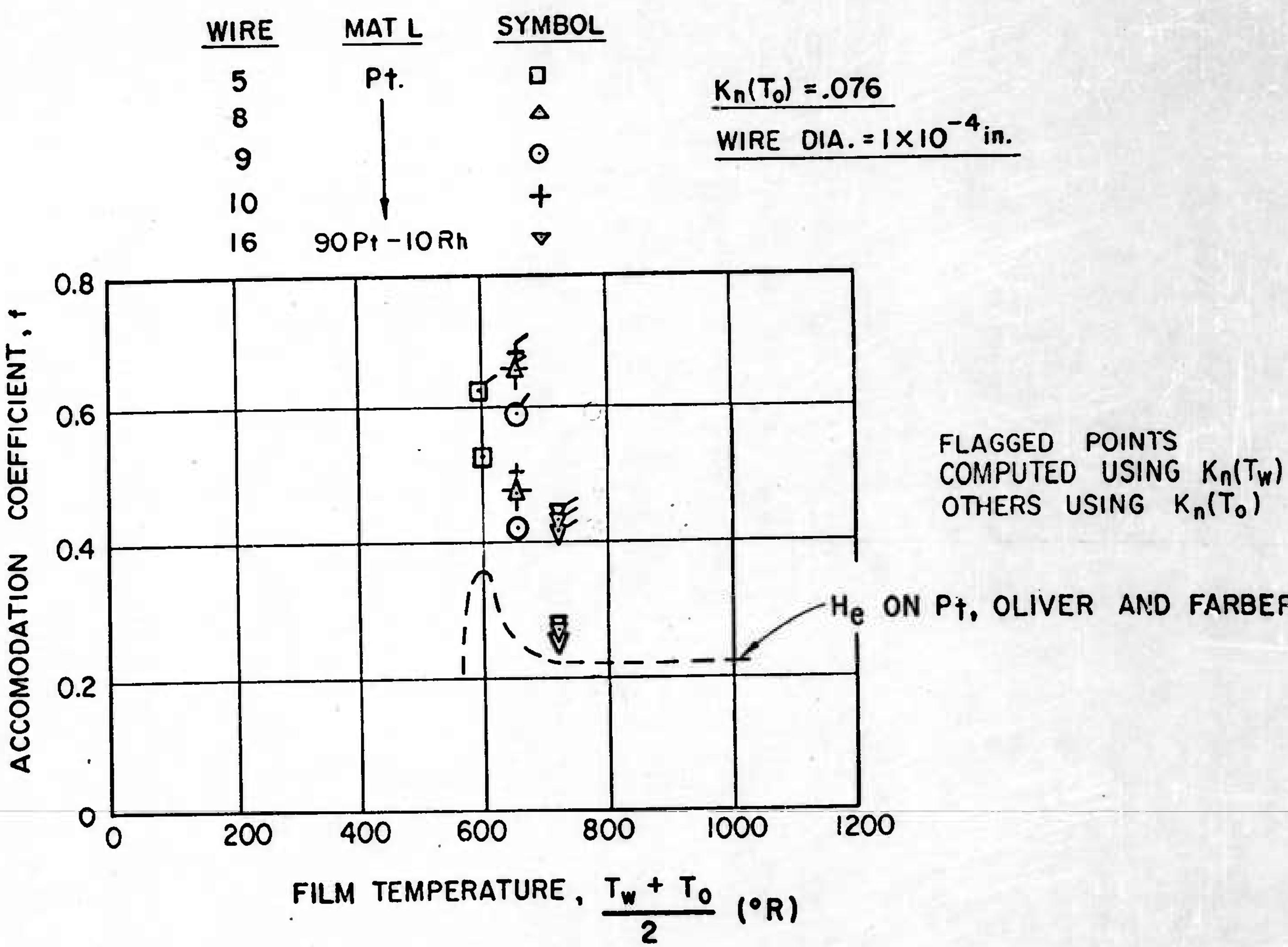

FIG. 9 - COMPUTED ACCOMODATION COEFFICIENT FOR He ON Pt.

IV KI-7

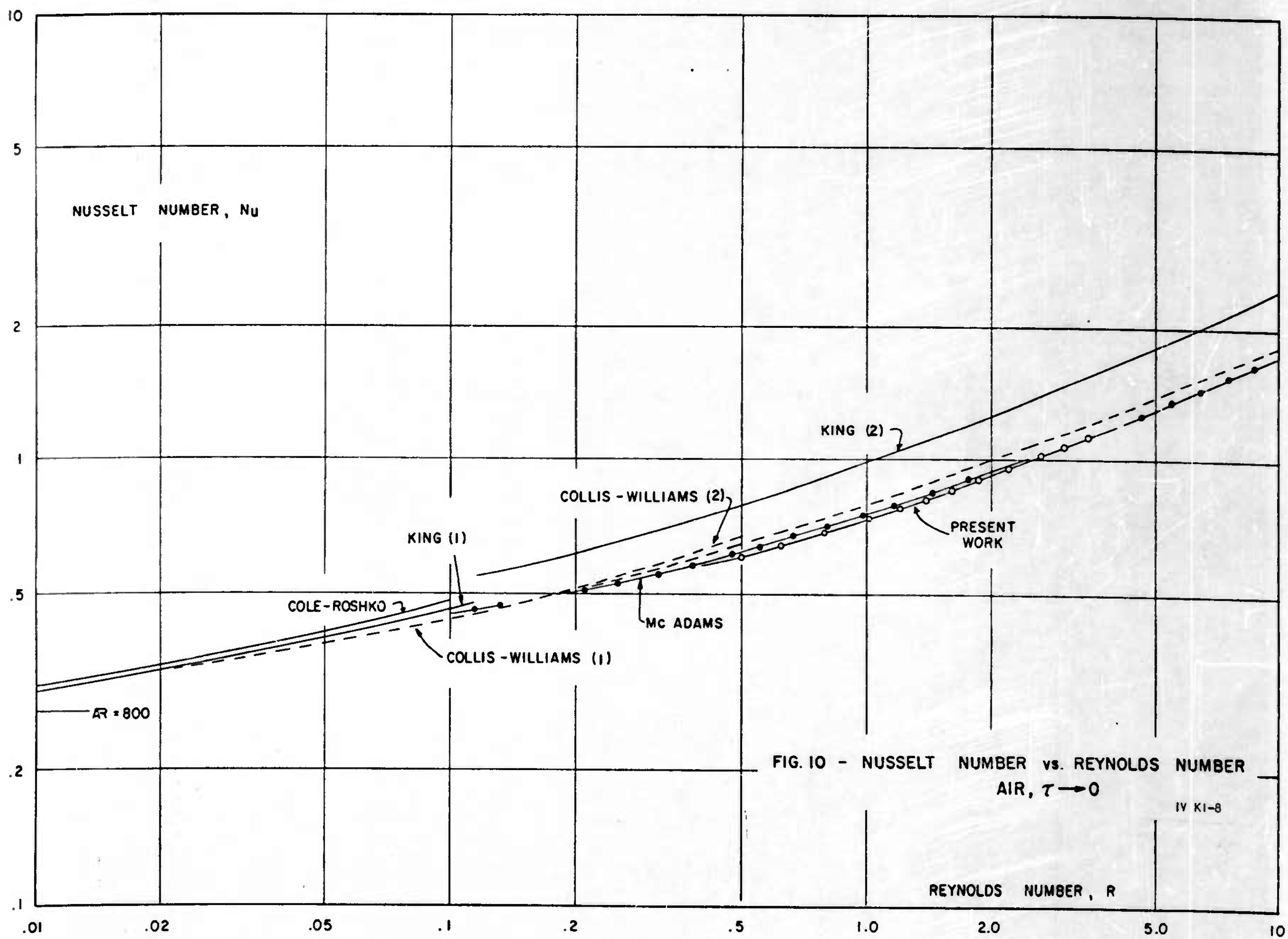

FIG. 10 - NUSSELT NUMBER vs. REYNOLDS NUMBER
AIR, $\tau \rightarrow 0$

IV KI-8

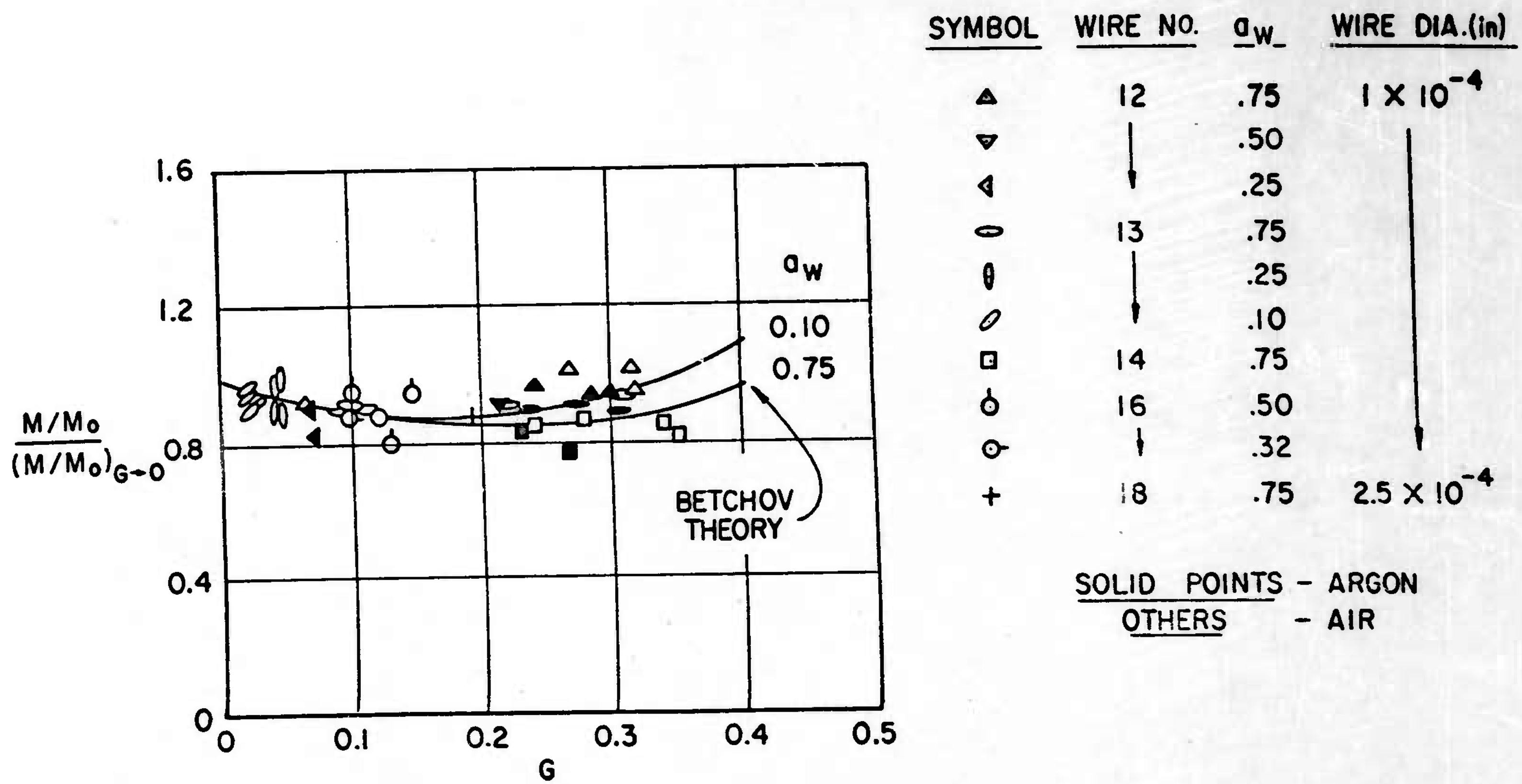

FIG. 11 - COMPARISON OF MEASURED TIME CONSTANTS WITH THEORY OF BETCHOV.

IV K1-9